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NEW CLASS OF HYBRID THIRD-ORDER APPROXIMATIONS FOR FRACTIONAL DERIVATIVES AND APPLICATIONS

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Higher-order approximations for fractional derivatives (FDs) and integrals have recently been of great importance for numerous real-world applications in many branches of science and engineering, including fractal phenomena, anomalous diffusion, viscoelasticity, and biological population models. In contrast to the classical derivatives, FDs play a leading role in describing memory and hereditary properties of materials and processes; for example, the anomalous diffusion model is described by the continuous random walk in a stochastic process. The shifted Grünwald approximation (ShGA) with shift $r = 1$ is well-known to be of first-order accuracy for approximation of FDs. Recently, a second-order accurate Grünwald type approximation with arbitrary shift (ShGTA) was constructed in the literature and proved to be reliable with shifts $r = -1, 1$ for stability and consistency with numerical verifications for FDs of order $\alpha \in \{1, 2\}$. In this study, a new class of hybrid third-order approximations was established. The approximation formula for this class was obtained from a hybrid convex combination of one ShGTA with shift r and a convex combination of two ShGAs with two distinct shifts r_1 and r_2. This class includes 18 different third-order approximations corresponding to a shift combination (r, r_1, r_2) with $r, r_1, r_2 \in \{-1, 0, 1\}$. We apply them for the fractional boundary value problem of order α. Numerical test examples are accomplished to verify the accuracy and convergence order of each approximation. The numerical results confirm that the accuracy and the convergence order of each approximation except the case where $r = -1$, $r_1 = 1$ and $r_2 = -1$. The analysis of stability properties of the class is the next step of our study.

Keywords: Fractional derivatives, Second-order approximation, Shifted Grünwald approximation